The 3rd XI batting order: an unusual tail

Summary

There are layers of myth and confusion to discard. We must return to first principles, which for the 3rd XI appear to all good judges to be:

1. Eleven players will bat. The proportion of 3rd XI first innings in which fewer than nine wickets fall is negligible. In 2012 only the game against bottom side Hanley and Upton featured a first innings where the Green set a target, and only the win against next-to-bottom side Martley featured a successful chase instead of a fatal collapse.
2. None of the batting order “strategies” proposed and tried by skipper Dick Brown has been shown to bring a statistically significant improvement. We include here such bright ideas as “lengthening” the batting by alternating strong and weak players, “kickstarting” the innings with a violent but high-risk batsman at 1 or 2, “holding back” senior or sensible players “in case wickets fall quickly”, and of course the catch-all tactic usually signified in the dressing room by “We'll try something different today”.
3. Corollary: all the above are on average worse than the ordinary method of deploying a somewhat senior opening pair (see under Hill, Ratcliffe, Baddeley) followed by a more aggressive middle order (White, Thompson, Alamutu, Anshuman), with Brown himself no higher than 7.
4. The essentials are captured by two lines of an old essay on team selection [1]:

> *Yet arts that thrive at Number Five*
> *Don't take at Number One*

But there is more. When thou hast done, thou hast not done (J Donne); when all this is accepted, there remains a point that, as far as is known to date, has no simple answer: *at what number should Dave Price bat?*

The issue is deceptive, and its difficulties only increase with repeated examination; it may be, literally, undecidable. That conclusion was reached by respected colleagues years ago, and has been vigorously defended. My own view, for what it is worth, is that a correct solution may well be achieved in future, but is not at all likely in the *near* future – say, this season or next. And a solution, although it will very probably contain elements of the arguments outlined below, must almost certainly go beyond them in ways that are still entirely unclear.

Readers who are not familiar with modern mathematical philosophy and its apparatus should take heart. If baffled they are not alone, and there are several excellent published accounts that treat essentially the same material from different angles [2], [3].

[1] T Hood, “Number One”, in *The Comic Annual*, 1830.

[2] D Hofstadter, *Gödel, Escher, Bach: an eternal golden braid*, 1979.

[3] R Penrose, *Shadows of the Mind: A Search for the Missing Science of Consciousness*, 1994; http://www.calculemus.org/MathUniversalis/NS/10/01penrose.html.

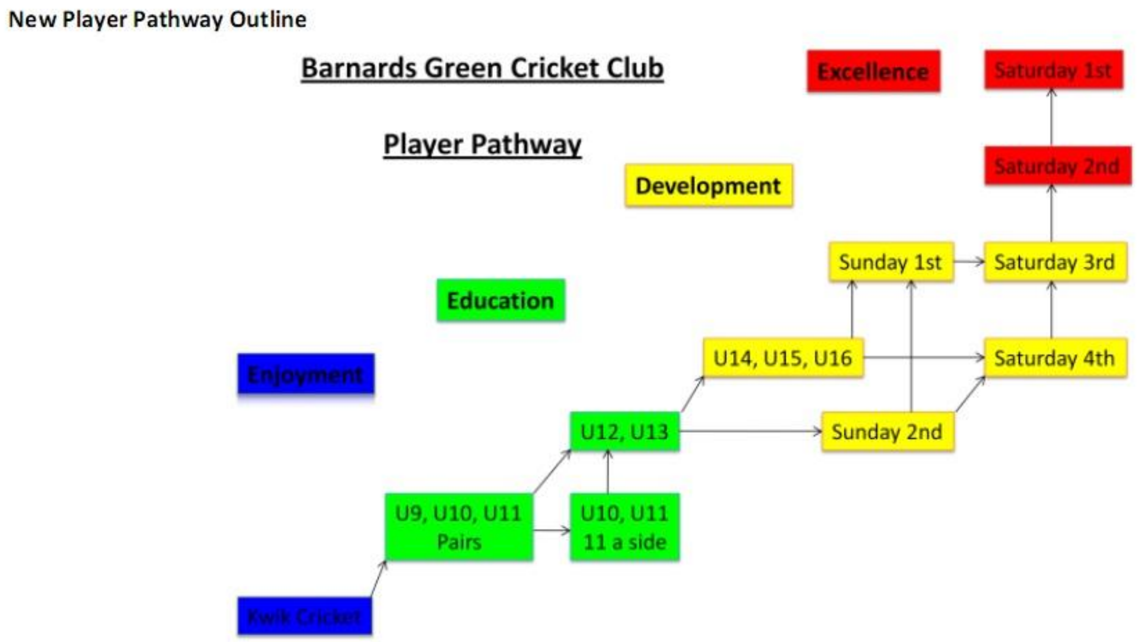
Technical appendix: optimally ordered lists and a Barnards Green Gödel sentence

This can be only a short survey of a vast, fascinating topic. To all the Saturday XIs we echo the authors of a noted text from half a century ago [1]:

We can, we feel, recognize four large groups of readers, as far as attitudes and backgrounds are concerned...We say to the members of each group: "Though it may seem to you that it is not so, we have tried to consider you in writing this account. Matters are not simple, but we could have easily made them more complex".

Now at first sight the problem does appear simple to state and solve. The "batting order" is certainly a well-defined mathematical object, which we can express as an *ordered list*, or a one-to-one *mapping* between eleven players and the integers 1 to 11 inclusive. The *players* in question are *selected* each week from a finite *pool* of "available" club members.

We do not need to specify at this stage any particular algorithm for the selection process; indeed different clubs, or different match-days at the same club, are known to have quite different processes, "committees", timescales and so on. We need only be confident that, whatever the precise details, the process *is* expressed or captured by some formal logical *system*. This is certainly our everyday understanding of the behaviour of cricket clubs, and for BGCC it is strongly supported by various statements and "policies", for example the *Development and Business Plan 2012-2016* which states that "Selection is carried out by the captains with the Senior Cricket Manager acting as Chairman", according to a clearcut but puzzlingly one-directional *Player Pathway* flow diagram:



Readers versed in the mathematical and popular literature will recognise that, in effect, we are requiring the selection process to be described by a *Turing machine*; or more explicitly that the overall problem is represented by two stages (first *selection*, and then *ordering*), where this two-stage process can be represented by a single Turing machine - which should not be remarkably more complicated than one representing either stage alone. (The two stages are consecutive and simply connected, in that first the four Saturday XIs are selected in *non*-ordered lists, and then the lists are passed to the second stage of ordering). A Turing machine may be imagined to accept an *input*, to perform a series of simple operations according to its previously established logical rules, to *stop*, and (upon stopping) to provide an *output*. In our case the input can be assumed to be the pool of available players, and the output is, in part, the list of eleven selected 3rd XI players in batting order. The machine stops if its internal algorithms ("programs", or committee/captain/selection/ordering "procedures") produce such a list of exactly eleven available players in a defined batting order that obeys some minimum criteria of "soundness" or "consistency". It may stop in various other circumstances, such as when the inputs are "invalid" (e.g. the pool is smaller then eleven). And in some cases it may not stop at all.

For any *particular* input we expect that the machine will definitely either stop or not stop, and we may ask which of these two possibilities is true. That is, knowing the particular input (the pool of players), and knowing the full details of the internal processes (Turing machine), we ask *will the machine stop?* It was the spectacular, unsettling achievement of the Austrian mathematician Kurt Gödel to show that, quite generally, for any reasonably extensive formal system (or set of procedures), there are inputs for which this question is *undecidable*. We may put this another way by saying that there are propositions, expressible within our (selection) system or language, that cannot be shown to be either true or false if we rely only on the rules of that system – where these are rules which we *know* to possess logic consistency (what Gödel called *Widerspruchsfreiheit*, more literally "freedom from contradiction") .

The implications of this were deeply shocking within pure mathematics and the philosophy of logic, but they have a perfectly general reach and it is time to consider other, and in our case sporting, fields. To put this in perspective: Gödel's original conference discussions and published paper coincided with the 1930 Ashes series in England (after which an algorithm of unknown complexity was used to award £10 bonus per Test to each English professional, but a fixed sum to the Australians). Albert Einstein had already been saying for several years to Born, Bohr and other colleagues that *God does not throw dice*, or German words to that effect; during his appreciative encounters with cricket games in early 1930s England he would have phrased this as *God does not toss coins*. Some decades then elapsed while mostly fruitless or ill-founded theories were elaborated and pursued to various dead ends. The relevance of cricket to the theoretical foundations of logic was obscured by wartime interruptions and vacillating discussion of the LBW law. Yet for the last twenty years a decidedly sharper focus has been evident, and it is not too early to claim that a wide consensus has now emerged.

The key factor in this breakthrough – it happens by chance to involve a dedicated British team, though greatly aided by international collaborators – is the *Price paradox* or, as it was originally called, the principle of the *excluded middle order*. Without going into excessive technical detail, we will summarise the historical steps and main results.

Until roughly 1988 it was believed that the two aspects of the batting-order problem satisfied simple (and, from the point of view of the captain, most desirable) conditions:

I. For each place in the order (1 through N) the players could be unambiguously ranked in order of suitability; **and**

II. For each player, the player's suitability for the places (1 through N) in the order could be unambiguously ranked; **and**

III. For each player, the graph of suitability against batting position (1 through N) would show a *single* peak.

Some illustrations may help. This is a toy[1] example for a two-player side:

Example 1	Player 1	Player 2
Batting Position no. 1	2	1
Batting Position no. 2	1	2

From left to right, we have two selected players; from top to bottom, we have the two available positions in the batting order. Each player, for the sake of argument, is allocated 1 + 2 = 3 "merit points" representing his or her "suitability" for each position; the larger the number, the stronger the claim to that position *in the view of the captain* (meaning, for present purposes, according to our Turing-type computations).

Evidently one can extend this type of structure – a square diagonal matrix, in the jargon – to arbitrary size and in particular to an 11 x 11 matrix:

Ex. 2	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
BP1	11	9	7	5	3	1	1	1	1	1	1
BP2	10	11	9	7	5	3	2	2	2	2	2
BP3	9	10	11	9	7	5	4	3	3	3	3
BP4	8	8	10	11	9	7	6	4	4	4	4
BP5	7	7	8	10	11	9	8	6	5	5	5
BP6	6	6	6	8	10	11	10	8	6	6	6
BP7	5	5	5	6	8	10	11	10	8	7	7
BP8	4	4	4	4	6	8	9	11	10	8	8
BP9	3	3	3	3	4	6	7	9	11	10	9
BP10	2	2	2	2	2	4	5	7	9	11	10
BP11	1	1	1	1	1	2	3	5	7	9	11

Example 2 satisfies our condition II (because we assumed it did by using these 1+2+3+4+5+6+7+8+9+10+11 = 66 points for each player) and also condition III (each column falls away "monotonically" from a single peak of 11 points). But it clearly fails condition I, and in fact none of the 11 matrix rows (one for each batting position) contains all the integers 1 to 11 once and once only.

During a well-fuelled international conference in the late 1980s, by a *tour de force* remembered with awe, Lewis succeeded in showing that conditions I, II and III could not all be satisfied for a team of 3, never mind one of 11. Not all his colleagues were convinced, and the meeting broke up in confusion. With a research community polarised along regrettably national lines, and big-ticket UK funding diverted to "initiatives" on Sudoku, many despaired. Our doughty

[1] * Or not so toy; some profound results are emerging from the analysis of single-wicket competitions. See M Ashdown, "The single combat in certain cycles of English and Scandinavian traditional romance", Modern Language Review XVII (1922), 113-130.

skippers stood accused of parochialism and blind trust in *ad hoc* methods. Their expertise covered many fields, but not optimal-list recursive algorithms. Surely disaster *must* ensue, late or soon, with three (or no) batsmen attempting to take strike at the same time. But somehow our selectors and captains continued to run on plain British common sense: an eleven-player list along the general lines of Example 2, with each player sitting more or less comfortably, appeared every week. Occasional wobbles, or full-scale collapses, were dismissed as the effects of improper prior information or random fluctuations; “Where's the fun in winning *all* the time?” was a common cry.

This neat utopia was upset by the fundamental theorems of Lewis and Lamb, who showed that in principle a player could be highly suited to an opening spot (1 or 2), and highly suited to a tail-end spot (say 9, 10 or 11), but hopeless in between – hence the phrase used above, *excluded middle order*. Other researchers, following this lead, soon found examples of players who were best (or worst) placed at an odd number, an even number, a prime number (*Primzahl*), and so on. The situation took a nasty turn when **David Price** appeared on the playing scene. Not necessarily able to explain his own phenomena, Price rapidly convinced colleagues and neutrals of a remarkable fact: he naturally did not and could not settle in any batting spot, regardless of the players around him, the match opponents or conditions, and whether he was earmarked for a spell of scoring or umpiring before or after his innings. Bafflement spread. Price's comments such as “I couldn't help it” after a cheap dismissal, or “Just had a swing” after a series of wholly unexpected and winning boundaries, shed no additional light.

As so often happens in dark times, a breakthrough was at hand. We cannot follow here all the real-life intricacies and functional dependencies when these dancing numbers, in their mysteriously arranged rows and columns, interact under pressure of conflicting evidence about pitch conditions, the team's mental state, and the likely tea menu. We cut to the chase: finally, in a prodigious demonstration of mathematical logic, Brown (2011) has shown that under rather general conditions the above question is indeed *undecidable*, and with a crucial twist: if Price is in the pool, the question “Is Price in the team?” is undecidable; moreover, and somewhat unexpectedly, Brown shows that *if* Price is in the team, then the question “What is Price's batting number?” is *also* undecidable.

These conclusions are, of course, not necessarily welcome or comfortable, and they were soon attacked on various grounds [2]. Again it is not possible in our limited space to cover the whole controversial history, but by and large the objections fall under the following headings, and in every case I believe they can be satisfactorily rebutted.

Q1. The selection pool is not a well-defined object

Supporters of this objection tend to mean in practice that the pool is not a finite set, or at any rate (even if infinite) not a set of cricketers that can arise by well-specified finite methods. Such an objection, although familiar from the constructivist squabbles of twentieth-century mathematics, appears initially absurd; surely it is just obvious that the pool or set of available players is finite, countable, and indeed a perfectly well-defined entity consisting of a (not particularly large) integer number of eligible players?

Yet a moment's reflection, and even more a few minutes' examination of the match scorebooks, may suggest that some care is needed. Examples abound of match-day teams that bear no resemblance to the initial “selection”: friends, relations, and in some cases entire strangers appear for no clear reason except that certain players have dropped out because of injury, alcoholic poisoning or inability to interpret a road map. It is not generally true (though it is sometimes locally true in the most senior teams) that players must be chosen from a list of eligible, paid-up, “registered” cricketers – in effect, from a possibly large but finite “pre-selection” list.

Despite determined attempts to exploit this looseness in the problem's statement, most researchers agree that the pool *is* (for (all (practical purposes[2])[3])[4] finite and well-defined. If necessary, they are prepared to buckle down and start from the entire non-juvenile human population of our planet, including in the "downselection" process every possible vicissitude of illness, mistaken identity, and match-day travel detour: then the pool certainly has an integer number of qualified candidates, that number is currently not more than seven billion, and we can proceed.

This leaves a somewhat sour taste of inefficiency but it seems to dispose of the formal objection, except for hardbitten constructivists who demand that this "maximal set", and all steps leading from it through smaller sets to the final selected team, must be capable of absolutely explicit description in finite terms (and, in particular, within the time interval between Tuesday's selection meeting and Saturday lunchtime, which may include changes in availability at either end of the age range). Nonetheless we will have reason to return below to certain difficulties implied in such a brute-force computation.

Q2. The selection process contains an essentially random or non-rational element

On the face of it this objection is powerful, since the premise is obviously correct and nobody would claim that selectors act rationally. However, to the surprise of most ordinary people and even many experts, the introduction of non-rational or random elements does *not* affect the issue. See Penrose (1994).

Q3. Similarly, the choice of batting order is essentially random or non-rational

The point here is not that the original algorithms have non-rational or random elements. If they do, the objection fails for the same reasons as in Q2. But what if an originally valid list, resulting from the correct operation of the Turing-machine procedures, is subsequently altered, up to and including the instants when particular batsmen are asked to pad up or enter the field of play? We can address this in two ways.

First, if the extra steps or alterations are themselves algorithmic, there is no difficulty in principle if we *include* them so that we now have a "superprocedure" or second-order selection, whose function is again that of a Turing machine which stops or does not stop. If our critic objects that the extra steps, or extra morsels of information or rumour used by an indecisive captain, are *not* definably "algorithmic" (in the sense of being capable of incorporation in a Turing machine), this in effect concedes the main conclusion about noncomputability that we are trying to establish. See Penrose on this, and also on the sometimes misunderstood point that it is *cheating* to introduce fresh rules that lie *outside* the algorithm and then use the results to claim access to fresh inferences (such that a previously undecidable proposition is now indeed true or false). For *any* such "superprocedure" we can derive its own, undecidable, Gödel sentence.

Second, a *reductio ad absurdum*: one can always imagine a case where everyone else has batted and therefore only Price remains; this is an extreme example of the time-dependence (nonstationarity) built into the real-life problem. There are analogies with the celebrated paradoxes of "unexpected hanging" or "surprise examination" (Franceschi 2005): instead of being told that he will be executed on one of the following days (but that he will not know *which* day until he awakens on the day itself), the batsman is told that he will bat in some position N (but is not told *which* position until the $(N-2)^{th}$ wicket falls). The literature contains numerous detailed and

[2] PP

[3] Abbrev. APP

[4] Often abbreviated FAPP

confusing treatments within the formalisms of classical probability and Bayesian analysis, to which more eager readers are referred. It is of course likely that the Price paradox, in the standard form that I am discussing, will be joined (as this research field evolves) by a number of interesting related examples.

Q4. Are the two stages in fact computationally separable?

Or do they interact, so that for instance the team selection cannot proceed without consideration of the batting order? Evidently, if this objection has teeth, we may face a kind of chicken-and-egg situation. But Brown's very clear exposition established that the “procedures” of selection *can* be extended straightforwardly to include recursive loops, dithering, referring to sub-committees or higher committees, and so on.

Q5. The rules for choosing a batting order are not logically consistent

Note that we are talking here of the logical consistency or *Widerspruchsfreiheit* of our underlying formal system, not of the *merits* (if any) of the reasoning (if any) followed by the selectors and captain. The difference is important and has often been misgrasped.

Hill, inevitably, had an opinion (2010). It is the usual practice to select a team and then its batting order by giving due weight to the blend of player attributes (bowling, batting, fielding, motorised transport and so on), and in particular by assuming an unbiased toss. Recent seasons place a question mark against this evenhandedness; Brown's record in League games is appalling (see *bgcc3rds2011* at www.barnardsgreen.com/reviews.html), and one might expect the selectors to adopt, or at least give fair trial to, a policy that recognised the fact. Since the published 3^{rd} XI often contained no more than two recognised bowlers and no more than three recognised batsmen – though the interpretation of these labels is itself almost insuperably challenging – Hill voiced serious doubts about whether such policy and “algorithms”, even if intelligently planned, had been or could ever be correctly implemented.

With typical thoroughness he then raised the important but overlooked problem of the *senior all-rounder*: put simply, that tendency of Brown (and not only Brown) to open with vigour and youth, while “holding back” his more stolid players “in case” the youngsters flop and a recovery is needed. This approach is fatally flawed, as was clear in advance and as events soon proved. Since all eleven players bat (see above), Hill at (say) no. 7 could expect to pad up for a stressful and prolonged rescue operation. He would then, after a sweaty hurried tea, be asked to bowl an extended uphill opening spell. This is by no means obviously better than batting high in the order for a leisurely hour or so, enjoying a well-earned rest, taking time over some delicious cakes and sandwiches, and returning to the fray with Lillee-like aggression. After three or four repetitions there was at last some sign that Brown understood this.

These are hefty criticisms, but we cannot conclude that the system is *logically* flawed; foolishness and negligence do not imply self-contradiction. It would, for example, be contradictory to demand a batting order that satisfied all three conditions I-III above; demonstrating this was the great Lamb/Lewis advance. Other contradictions can lurk more subtly: for instance, instructions to batsmen to “Go out there and do your best for the team” can and should in some cases be countered with “But I can do my best by remaining here in the pavilion; get a grip”. It should always be borne in mind that an extremely poor procedure can still be a valid one if it is not fundamentally inconsistent. Even so, we note that several studies on the uses of *paraconsistent* logic in cricket tactics (Priest 2008, see http://plato.stanford.edu/entries/dialetheism/) are in progress. The ability to entertain a proposition and its negation, sometimes repeatedly before breakfast, has a sound pedigree.

Q6. The time required to perform the computation (i.e. select and order) is ill-defined or infinite

It is perhaps appropriate to point readers at the large literature on supermarket shopping algorithms, where an underlying problem of considerable similarity has been explored and several asymptotically optimum strategies already exist.

Enough. This recent British team effort has delivered the best and most reliable account so far, and it is strongly on Gödel's side. That might appear to be the end of the matter – the question as posed (which it is on most Saturdays during the season) is indeed strictly undecidable and we had better get on with our lives. Why then did I talk above about a different and "correct" solution in future?

In fact there may *still* be a potential loophole in the argument, connected in some way yet to be fully determined with the fact that *some players, but not all, are equal*. In popular technical language, there are questions about *fungibility* and *exchangeability* that remain unsettled. For example there is a sense, and it may be crucial, in which the *captain* of a side is one of the side – obviously – yet also, or alternatively, a player apart from the rest, with "the capacity to step back from the hurly-burly of the group" (Brearley 2001). This double nature has proved exceptionally challenging to incorporate in any of the standard treatments, and it may well be that a new line of thought is required. Whether it makes any sense to attach the "captaincy" to a single player (thus accepting the difficult issue of duality), whether this must be done at a particular stage in the selection process, and whether we can escape certain complications by somehow distributing "captaincy" over two or more players, remain basic puzzles in the field. Matters are not helped by "degenerate" cases of identical twins, triplets etc. of equal merit whose batting positions can be, according to some researchers but not others, freely interchanged.

In this context we note that a whole subclass of the literature treats the interesting point that for many years normal English practice was to identify a captain first and then (with the captain possibly acting as part of the "selection committee") select ten further players from the larger pool or squad, whereas Australia tended to select eleven players and then choose one of them as captain. This has some claim to be a significant difference, and a formidable battery of authors have argued about whether the undecidability problem arises in the same way for the two countries, and whether the joint selection problem can be well-defined (even in principle) when they play each other.

Poggitt (1967) invented a twelfth or pseudo-contributing player (who represented the "captaincy" functions) and attacked the problem with exceptional brilliance; but, trying to eliminate the unwanted variable (rather as in the technique of Lagrange multipliers), he encountered bewildering and tenacious mathematical traps. Price himself, in his notably terse and inconclusive 2005 paper, proposed a model of non-interacting skippers, but this seems to have been ignored. Longmore's bold approach, around the same time, discussed some fashionable ideas of quantum computation, with multiple "copies" of the team evolving along different hypothetical paths. Dubbed by some "fantasy cricket" or "a problem in search of a harder problem", this led to extensive but unimpressive results and is now out of favour.

My **conclusion**, and my great hopes for the future, will not surprise. We have taken important steps forward. But much work remains. Our club faces *insoluble* difficulties...and that is why we set the pace in internationally respected academic circles, and in Worcestershire County League (Division Four) (1^{st} XI).

References

[1] R Blackman and J Tukey, *The Measurement of Power Spectra*, Dover / ATT 1958.

[2] D Price (after observing the weather, June 2012): "The cricket's off tomorrow".
S Hunt (*aggressively*): "On what grounds?" D Price: "On both of them".

Additional technical references

J M Brearley, *The Art of Captaincy* (1985; 2nd edition 2001).

K Gödel, "On Formally Undecidable Propositions of Principia Mathematica and Related Systems" (*Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I*), Monatshefte für Mathematik, 1931.

P Franceschi, "Une analyse dichotomique du paradoxe de l'examen-surprise", Philosophiques, 2005, vol. 32, pp[5]. 399-421.

H H (not D) Price, *Belief* (the 1959-1961 Gifford lectures) gives a useful, or at least long, discussion of self-verifying and self-falsifying beliefs, with applications to batting: "It is an empirical fact that suggestion, including self-suggestion, is sometimes an effective method of enabling people to do things which they could not otherwise have done".

Note added in proof

After the above was written, the final 2012 league fixture provided astonishing and ludicrous scenes. The situation now appears fluid and Price himself may have dramatically extended our subject. First, during the Old Vigornians innings, he worsened an existing muscle condition and called for an elastic support; one was fetched by the observant Brown (himself sidelined with a torn hamstring, and generously acting as umpire and teatime counsellor). But an incorrect (one-piece, non-fastening) support was supplied; the essence of the topological problem is non-commutativity, in that once Price's trousers were on (as necessarily they were for a 50-over outdoor session) he faced the problem of rolling or tugging the tubular support a considerable distance *under* them yet *over* his shoes and socks.

This was a novel challenge; Price did not baulk at it, but neither did he succeed. After some delay and several expletives, he abandoned the tube and called for a Velcro-fastening version, but the damage was done. Energetic fielding, supplemented by amusing gymnastics, had brought intolerable strain, and he retired from the field. The *ten*-man Green side continued bravely, and in fact had more success as they restricted OVs to 255 runs. After a sound tea (see the present author's *Cricket Teas*) Hill and Baddeley started promisingly (see above), and all results seemed possible during Hill's dogged 30-over stint, but when he was out – perhaps unable to believe that the original selection contained at least one player less fit than himself – catastrophic collapse ensued (see above).

The upshot was that OVs had 3 overs in which to take the last Green wicket, but *Price had left the ground*. He would have been listed as no. 11, but had eliminated himself from consideration. Does such self-removal count as part of the selection algorithm, or have 80 years of profound analysis been overturned in one brilliant stroke? More news from the frontiers of sport is expected as we go to press.

[5] not PP